A note on the scale evolution of the ETQS function $T_F(x,x)$

Andreas Schäfer[1] and Jian Zhou[1]

[1] *Institut für Theoretische Physik, Universität Regensburg, Regensburg, Germany*

Abstract

We reexamine the scale dependence of the ETQS (Efremov-Teryaev-Qiu-Sterman) twist-3 matrix element which has been studied already by the four different groups with conflicting results [1–4]. We find that we can in fact reproduce the results of [4] with the method [2] when we treat some subtleties with greater care, thus easing the mentioned conflict.

The ETQS matrix element plays an important role for the theoretical description of transverse single spin asymmetries (SSA) in the framework of collinear factorization. The control of Q-evolution is not only necessary to describe QCD dynamics correctly and to reduce the dependence of theory predictions on the factorization scale adopted, but such evolution equations give also insight into the functional form of higher-twist distribution functions. The idea there is to start evolution at a low scale and use the fact that the resulting form at a high scale is only little dependent on the low-scale input [5]. The latter is especially important in view of the limited experimental input of has to determine these functions.

The corresponding calculation was done in Refs. [1–4]. However, the result obtained in Ref. [4] differ from that derived in Refs. [1–3] by two extra terms. It was settled rather easily that one of these terms is due to a Feynman diagram which was missed in Refs. [1–3]. The second additional term in [4] which is proportional to $\delta(1-z)$ could not be reproduced by the other calculations so far. We show in this short contribution, how this term arises within the formalism of Ref. [2] due to a rather subtle fact related to the non-commutativity of a certain limit and a certain integration. We now hope to be able to do this calculation consistently in the light cone gauge.

The ETQS function T_F is defined through the following matrix element,

$$\int \frac{dy^-}{2\pi}\frac{dy_1^-}{2\pi}e^{ixP^+y^-}\langle PS|\bar{\psi}_\beta(0)gF^{+\mu}(y_1^-)\psi_\alpha(y^-)|PS\rangle \ = \frac{M}{2}T_F(x,x)\epsilon_\perp^{\nu\mu}S_{\perp\nu}\not{p} \tag{1}$$

In Ref [2], the light-cone gauge ($A^+ = 0$) with the retarded boundary condition, i.e., $A_\perp(-\infty^-) = 0$ was chosen such that $T_F(x,x)$ can be rewritten as,

$$T_F(x) = \int \frac{dy^-}{8\pi^2 M}e^{ixP^+y^-}\langle PS|\bar{\psi}(0)\not{n}\epsilon_\perp^{\nu\mu}S_{\perp\nu}i\partial_{\perp\mu}\psi_\alpha(y^-)|PS\rangle\ . \tag{2}$$

To calculate the splitting function, one has to take into account the contributions from the operators $\left(\bar{\psi}\partial_\perp\psi\right)$ and $\left(\bar{\psi}A_\perp\psi\right)$, because they are of the same twist. We plot the Feynman diagrams contributions for the real gluon radiation in Fig. 1, where (a) is the contribution from the partial derivative acting on the quark field, and $(b-d)$ are those from $A_\perp$ contributions. Virtual corrections only contribute to the contribution proportional to $\left(\bar{\psi}\partial_\perp\psi\right)$. Their contribution is the same as for the quark self energy correction.

Following the procedure presented in Ref. [2], we perform a collinear expansion for the hard scattering part to calculate the contribution from Fig. 1(a). The linear $k_\perp$ expansion term combining with the quark field will lead to the quark-gluon correlation function

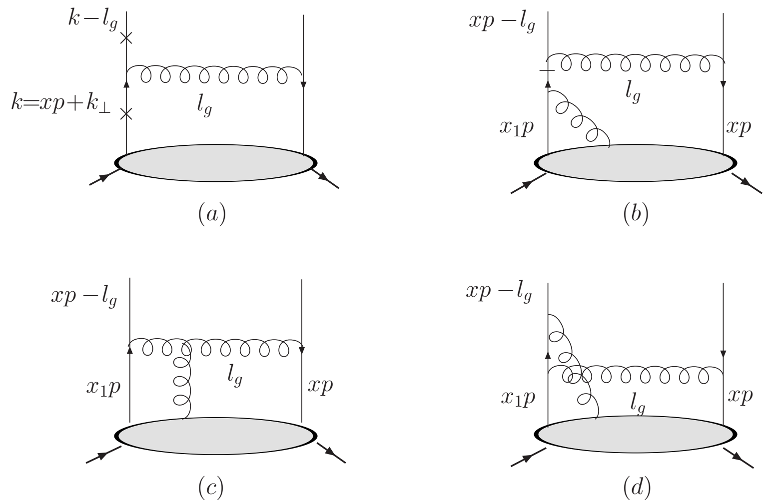

FIG. 1: Real gluon radiation contribution to the evolution equation for the ETQS function $T_F(x,x)$. Crosses in fig.(a) and horizontal bar in fig.(b) indicate $k_\perp$ flow and special propagator, respectively.

$T_F(x,x)$. In the collinear expansion in terms of $k_\perp$, we can fix the transverse momentum of the probing quark (l_q) or the radiated gluon (l_g), because of momentum conservation and we are integrating over them to obtain $T_F(x,x)$. We have also checked that they will generate the same result. Following the Ref. [2], we fix l_g in the collinear expansion to simplify the calculation.

For the $A_\perp$ contribution, we notice that $F^{+\mu} = \partial^+ A_\perp^\mu$ in the light cone gauge. Therefore, one can relate the corresponding soft matrix to the correlation function $T_F(x,x_1)$ in the following way,

$$\frac{i}{x - x_1 + i\epsilon} \int \frac{dy^- dy_1^-}{4\pi} e^{ix_1 P^+ y^-} e^{i(x-x_1)P^+ y_1^-} \langle PS|\bar{\psi}_\beta(0^-) \not{n} \epsilon_\perp^{\nu\mu} S_{\perp\nu} g F^+{}_\mu(y_1^-) \psi_\alpha(y^-)|PS\rangle$$
$$= \int \frac{dy^- dy_1^-}{4\pi} P^+ e^{ix_1 P^+ y^-} e^{i(x-x_1)P^+ y_1^-} \langle PS|\bar{\psi}_\beta(0^-) \not{n} \epsilon_\perp^{\nu\mu} S_{\perp\nu} g A_{\perp\mu}(y_1^-) \psi_\alpha(y^-)|PS\rangle \ . \qquad (3)$$

In above formula, the soft gluon pole appearing in the first line is generated by partial integration. The pole prescription has been determined by our choice of a retarded boundary condition. For the same reason, we have to regulate the light cone propagator in a consistent manner. The gluon propagator appearing in Fig. 1(c) in the light cone gauge with the retarded boundary condition is given by,

$$D^{\alpha\beta}(l) = \frac{-i}{l^2 + i\epsilon} \left(g^{\alpha\beta} - \frac{l^\alpha n^\beta + n^\alpha l^\beta}{l \cdot n + i\epsilon} \right) \ , \qquad (4)$$

where l is the gluon propagator momentum flowing out from the quark-gluon vertex in Fig.1(c).

We now deviate from the original calculation [2] in two ways:

i) in [2] the integral $\int_{x'_g} dx_g \frac{x'_g \delta(x'_g)}{x_g^2}$ was simply neglected;

ii) one of the two absorptive parts of the free propagator was not taken into account.

We will discuss next these two points in more details, arguing that the neglected contributions have to be taken into account. When computing the hard pole contribution from Fig.1(c), for the left cut diagram, one has

$$T_F^{(1)}|^{\text{hp}-\text{left}}_{\text{Fig.1(c)}}(x_B) \; = \; \frac{\alpha_s}{4\pi}\int_{x_B}\frac{dx}{x}\int_{x'_g} dx_g \frac{dl^2_{g\perp}}{l^2_{g\perp}}\frac{C_A}{2}\left[\frac{(x+x_B)(2x_g-x'_g)}{2x_g^2}\right]\delta(x'_g)T_F(x-x_g,x) \quad (5)$$

where $x'_g = l\cdot n/p^+$ with $x'_g = x_B - x + x_g$. By noticing that $\int_{x'_g} dx_g \frac{x'_g\delta(x'_g)}{x_g^2} = \delta(x_B - x)$, rather than zero, and summing left and right cut diagrams, one obtains,

$$T_F^{(1)}|^{\text{hp}}_{\text{Fig.1(c)}}(x_B) \; = \; \frac{\alpha_s}{2\pi}\int_{x_B}\frac{dx}{x}\frac{dl^2_{g\perp}}{l^2_{g\perp}}\frac{C_A}{2}\left[\frac{1+z}{1-z}T_F(x_B,x) - \delta(1-z)T_F(x_B,x_B)\right] \quad (6)$$

where $z = x_B/x$. The second term is missing in Ref.[2].

Next we discuss the second contribution which was overlooked in Ref.[2]. Since we work in the light-cone gauge with retarded boundary condition, the free propagator possesses two absorptive parts [6],

$$discD^{\alpha\beta}(l_g) = 2\pi\theta(l_g^0)\delta(l_g^2)\left[-g^{\alpha\beta} + \frac{2l_g^-(l_g^\alpha n^\beta + n^\alpha l_g^\beta)}{l^2_{g\perp}}\right] - 2\pi\theta(l_g^0)\delta(l_g^+)\frac{(l_g^\alpha n^\beta + n^\alpha l_g^\beta)}{l^2_{g\perp}} \quad (7)$$

In [2] only the first absorptive part was taken into account. In order to carry out the calculation in a consistent manner, one must include the contribution from the second part. However, if one still picks up the same imaginary part as we did above, this contribution will cancel between the different cut diagrams as it happens when both gluon lines go on shell. On the other side, the additional imaginary part may come from the artificial pole which appears in Eq.(3). Such pole-absorptive part combination gives the contribution,

$$\begin{aligned} T_F^{(1)}|^{\text{LC}-\text{left}}_{\text{Fig.1(c)}}(x_B) \; &= \; \frac{\alpha_s}{4\pi}\int\frac{dl^2_{g\perp}}{l^2_{g\perp}}\int_{x_B} dx\int_0^\infty dl_g^-\int_{x'_g} dx_g\delta(x_g - x'_g)\delta(x_g) \\ &\times\frac{C_A}{2}\left[\frac{2(2x_B-x'_g)(x_g+x'_g)}{2(2l_g^-x_B+l^2_{g\perp})x'_g} - \frac{2x_Bl^2_{g\perp}}{(2l_g^-x_B+l^2_{g\perp})^2}\right]T_F(x-x_g,x) \quad (8) \end{aligned}$$

Integrating over x_g, l_g^- and summing the two cut diagrams, we obtain,

$$T_F^{(1)}|^{\text{LC}}_{\text{Fig.1(c)}}(x_B) \; = \; \frac{\alpha_s}{2\pi}\int\frac{dl^2_{g\perp}}{l^2_{g\perp}}\int_{x_B}\frac{dx}{x}\frac{C_A}{2}\left[\int_0^1 dy\frac{2}{1-y} - 1\right]\delta(1-z)T_F(x_B,x_B) \quad (9)$$

Taking into account the contribution from the second part of the absorptive part, Eq.(20) in the Ref.[2] should be modified as follows,

$$
\begin{aligned}
& -\frac{\alpha_s}{2\pi}\frac{C_A}{2}\int_{x_B}\frac{dx}{x}T_F(x,x)d^2l_{g\perp}\left[\frac{\partial}{\partial l^\mu_{g\perp}}\hat{H}_0(xP,l_{g\perp})\right]\times(-l^\mu_{g\perp}) \\
= & -\frac{\alpha_s}{2\pi}\frac{C_A}{2}\int_{x_B}\frac{dx}{x}T_F(x,x)d^2l_{g\perp}\hat{H}_0(xP,l_{g\perp}) \\
= & -\frac{\alpha_s}{2\pi}\frac{C_A}{2}\int_{x_B}\frac{dx}{x}\frac{dl^2_{g\perp}}{l^2_{g\perp}}\left[\frac{1+z^2}{1-z}-\delta(1-z)\int_0^1 dy\frac{2}{1-y}\right]T_F(x,x)\ . \qquad (10)
\end{aligned}
$$

Collecting all pieces, we eventually arrive at the following scale evolution equation for $T_F(x,x)$,

$$
\begin{aligned}
\frac{\partial T_F(x_B,x_B,\mu^2)}{\partial \ln\mu^2} = & \frac{\alpha_s}{2\pi}\int_{x_B}\frac{dx}{x}\left[C_F\left\{\frac{1+z^2}{(1-z)_+}+\frac{3}{2}\delta(1-z)\right\}T_F(x,x)\right. \\
& \left.+\frac{C_A}{2}\left\{\frac{1+z}{1-z}T_F(xz,x)-\frac{1+z^2}{1-z}T_F(x,x)-2\delta(1-z)T_F(x,x)+\tilde{T}_F(xz,x)\right\}\right]\ , \qquad (11)
\end{aligned}
$$

which coincides with the result given in Ref. [4].

As shown above, the missing boundary term $-2\delta(1-z)T_F(x,x)$ appears to be a generic problem which might have far reaching consequences. In principle, all previous calculations involving hard gluon pole contributions might need to be reexamined. However, one can expect that the observed matching between the TMD factorization and collinear factorization at intermediate transverse momentum will not be affected by this extra term.

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